

# Computation of Signal Gain and Noise Gain of a SAR Processor

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**German Aerospace Center (DLR)**  
**Remote Sensing Technology Institute (IMF)**



Knowledge for Tomorrow

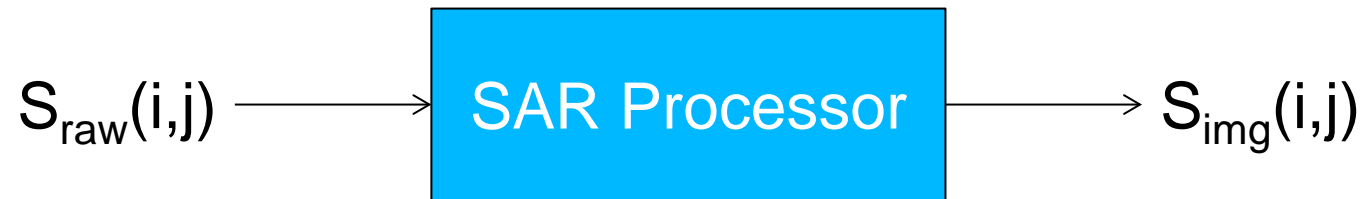


# Outline

- **Introduction**
- **Energy Normalization vs. Power Normalization**
- **Range (Pre-)Compression and Chirp Energy**
- **Azimuth Compression and Aperture Length**
- **Signal Gain vs. Noise Gain**
- **White and Colored Noise**
- **Tracking Signal and Noise Power through the Focusing Chain**
- **Conclusions**



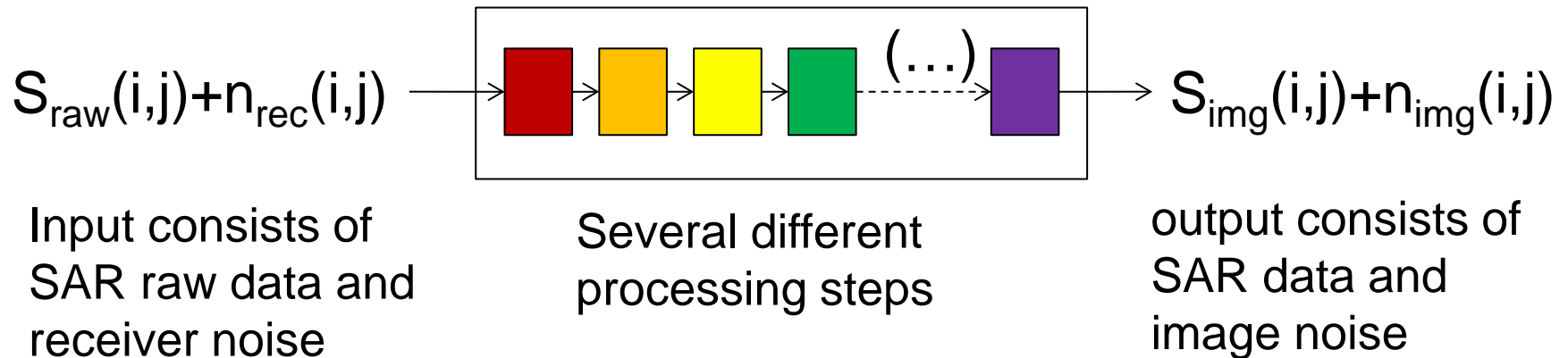
## Signal Gain of a SAR Processor (Simplified Model)



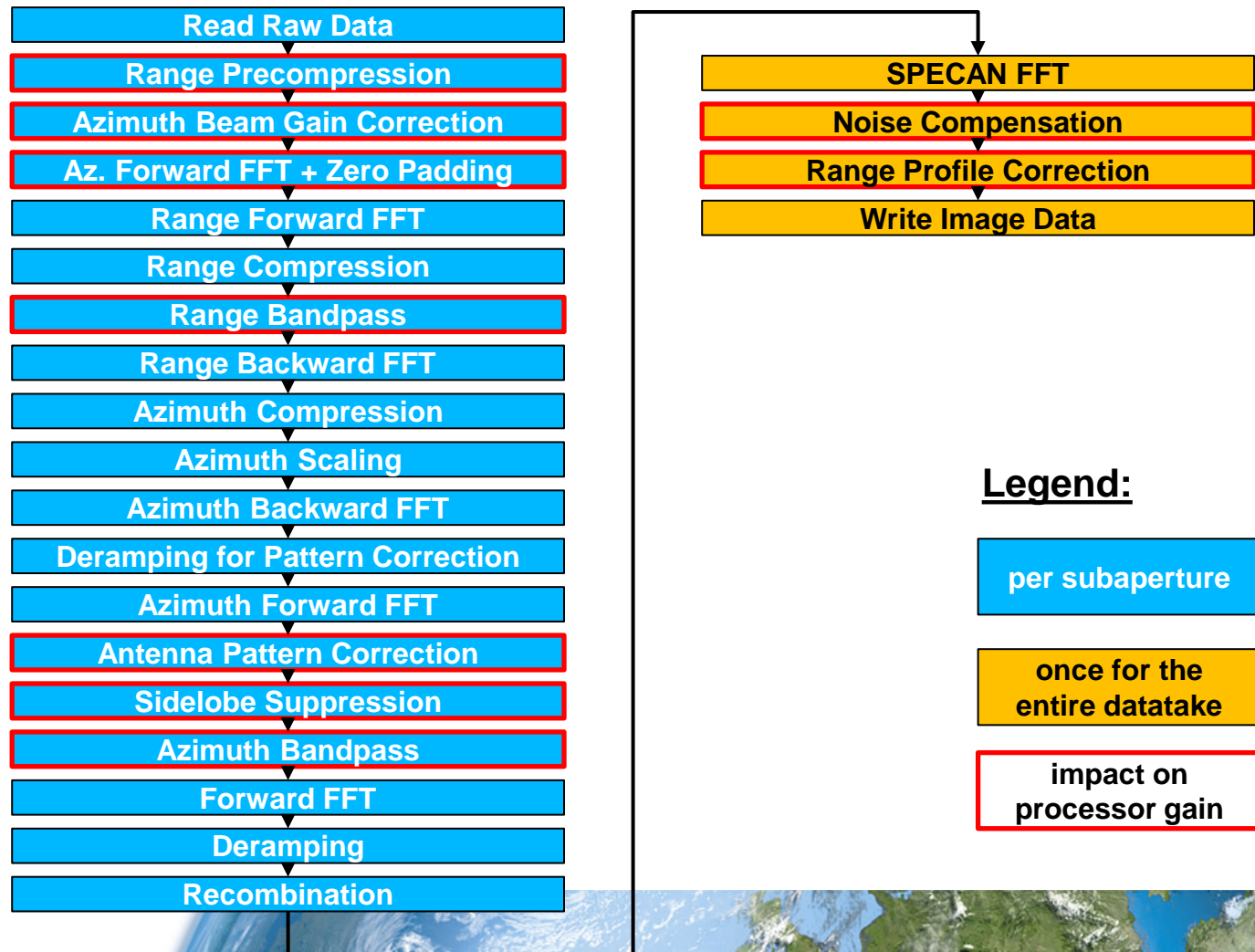
$$\text{gain} := \frac{\frac{1}{N'M'} \sum_{i,j} s_{\text{img}}^2(i,j)}{\frac{1}{NM} \sum_{i,j} s_{\text{raw}}^2(i,j)}$$



## Signal and Noise Gain (More Detailed Sight)

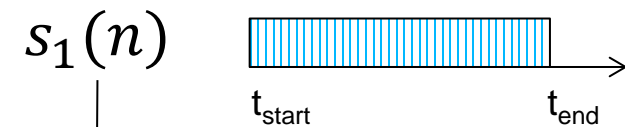


# Extended Chirp Scaling: Processing Chain for Sliding Spotlight Data

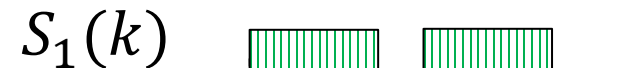


## Energy versus Power Normalization

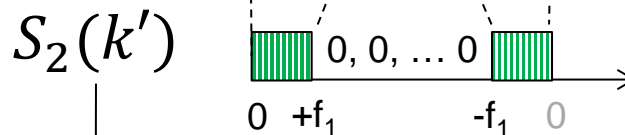
Changing the sampling rate of a signal:



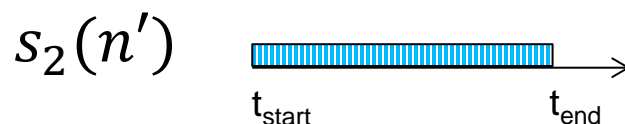
Transform to spectral domain



Spectral zero padding  
(**energy normalized**)

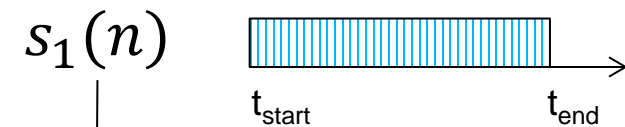


Transform back to time domain

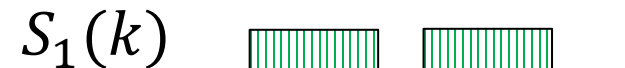


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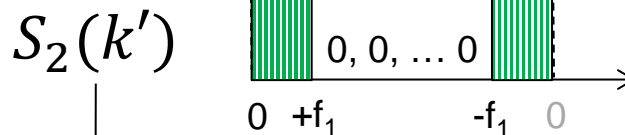
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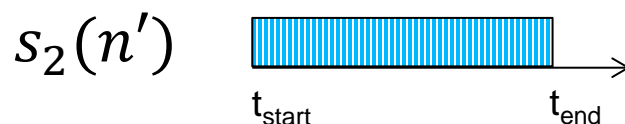
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# Range Precompression

$$s_{rc}(t, \tau) = s_{raw}(t, \tau) * r(-\tau)$$

Convolution with time reversed chirp replica in time domain





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Ditto. with normalization:  
Chirp energy occurs twice  
(in replica **and in the signal**)



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**A threshold avoids division by low values (or even zero)!**

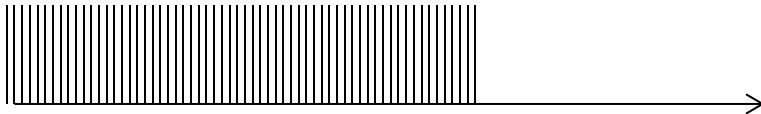


# Azimuth Compression

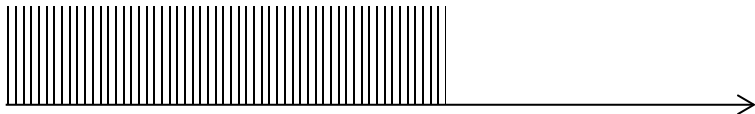
short aperture (e.g. ScanSAR):



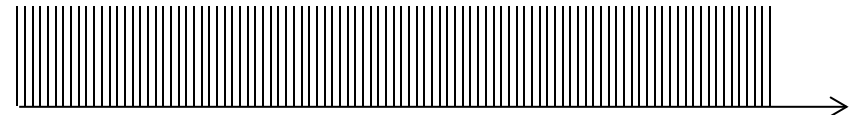
short aperture (e.g. near range beam):



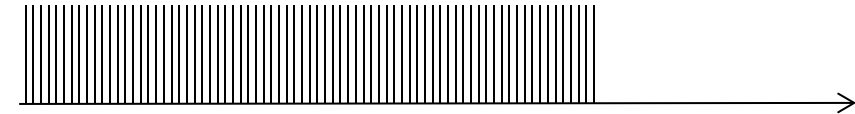
low PRF:



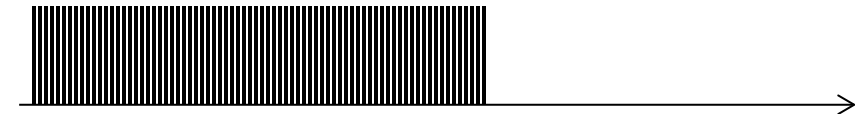
long aperture (e.g. Staring Spotlight):



long aperture (e.g. far range beam):



high PRF:



Received signal energy depends on:

- backscatter coefficient
- number of echo pulses in aperture

(wanted)

(unwanted)



# Azimuth Compression

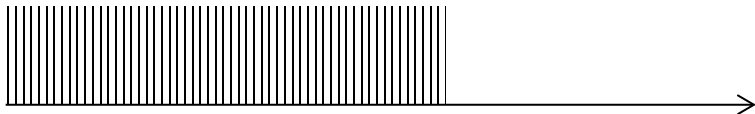
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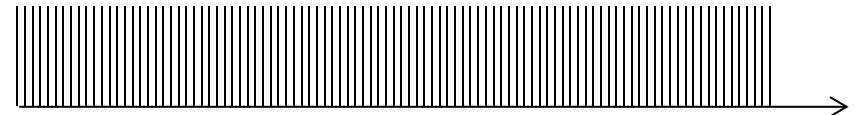
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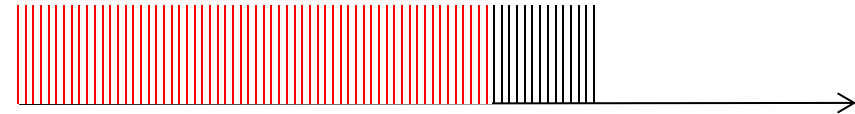
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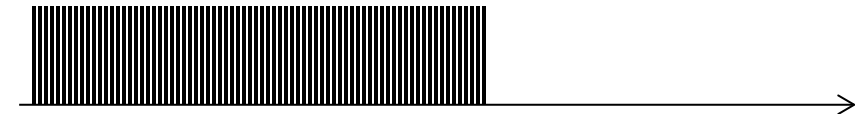
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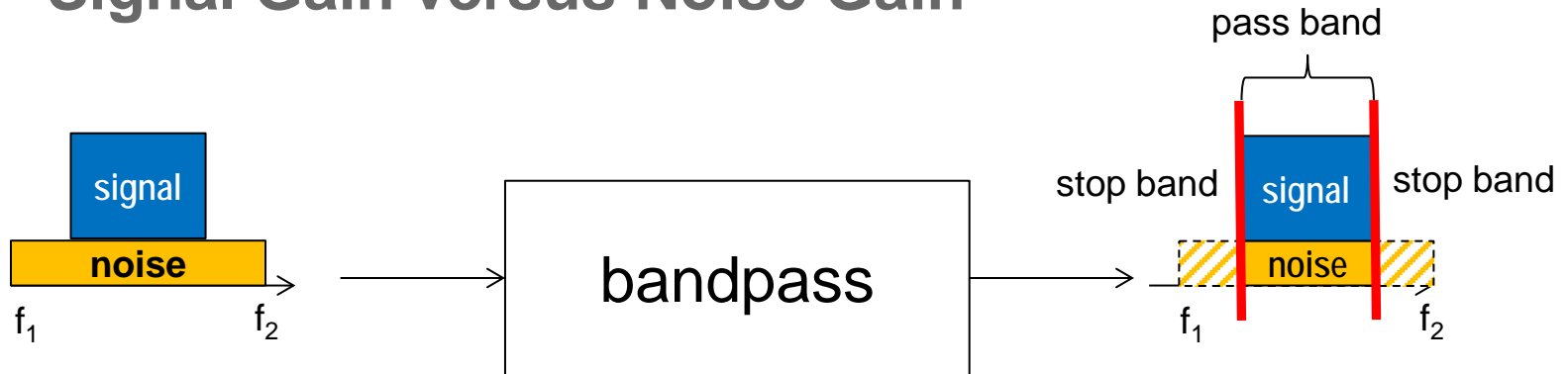
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## Signal Gain versus Noise Gain

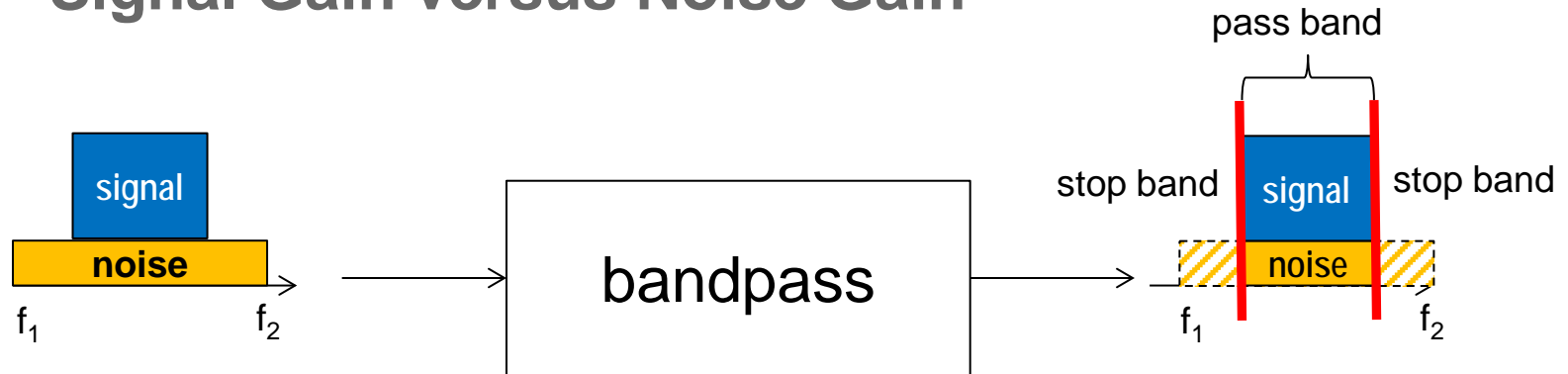


Input:  
narrow-band signal +  
wideband noise

Output:  
narrow band signal +  
narrow-band noise



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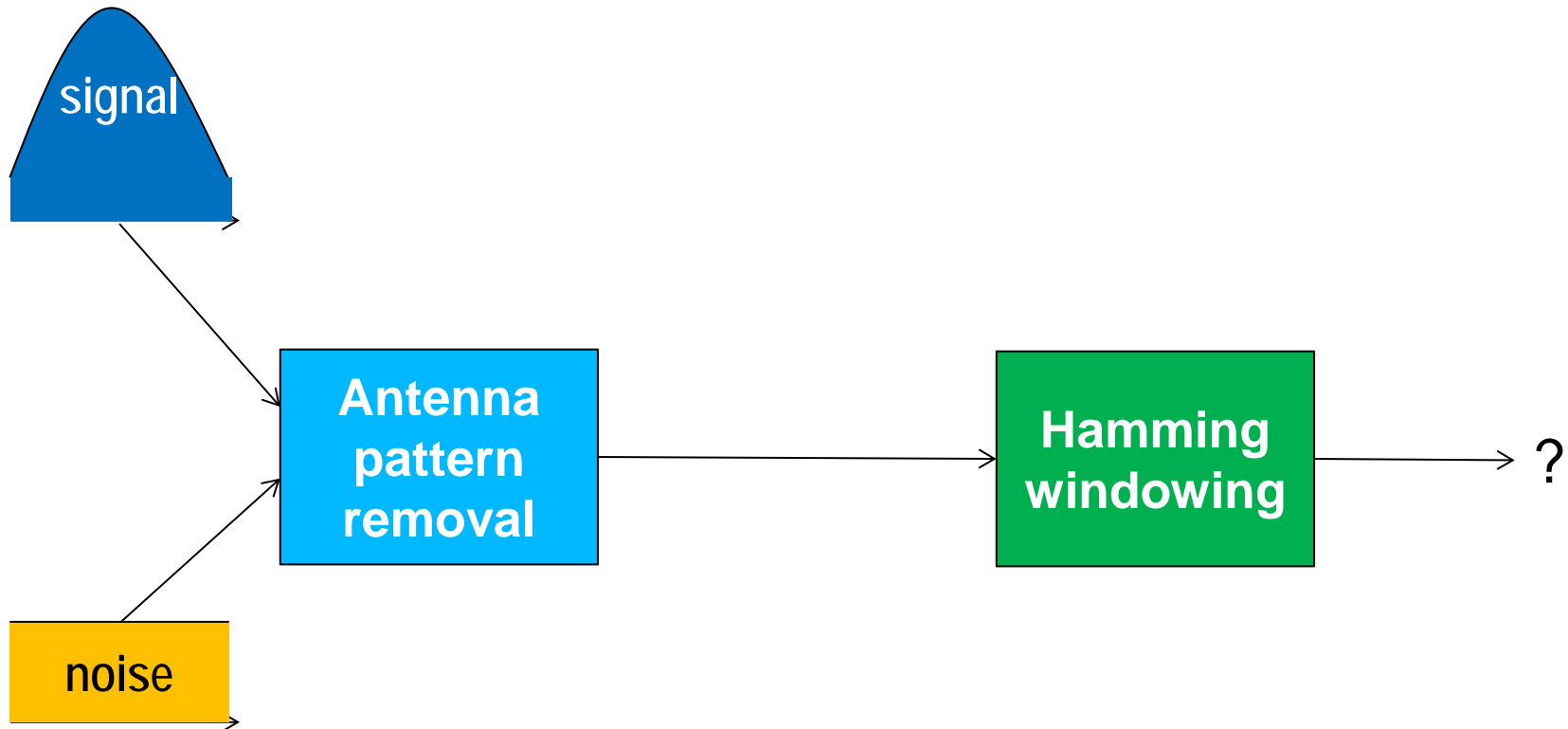
If bandwidth of signal and noise differs:

Different portions of signal and of noise energy are removed.  
Consequently, signal and noise gain of the filter are different.

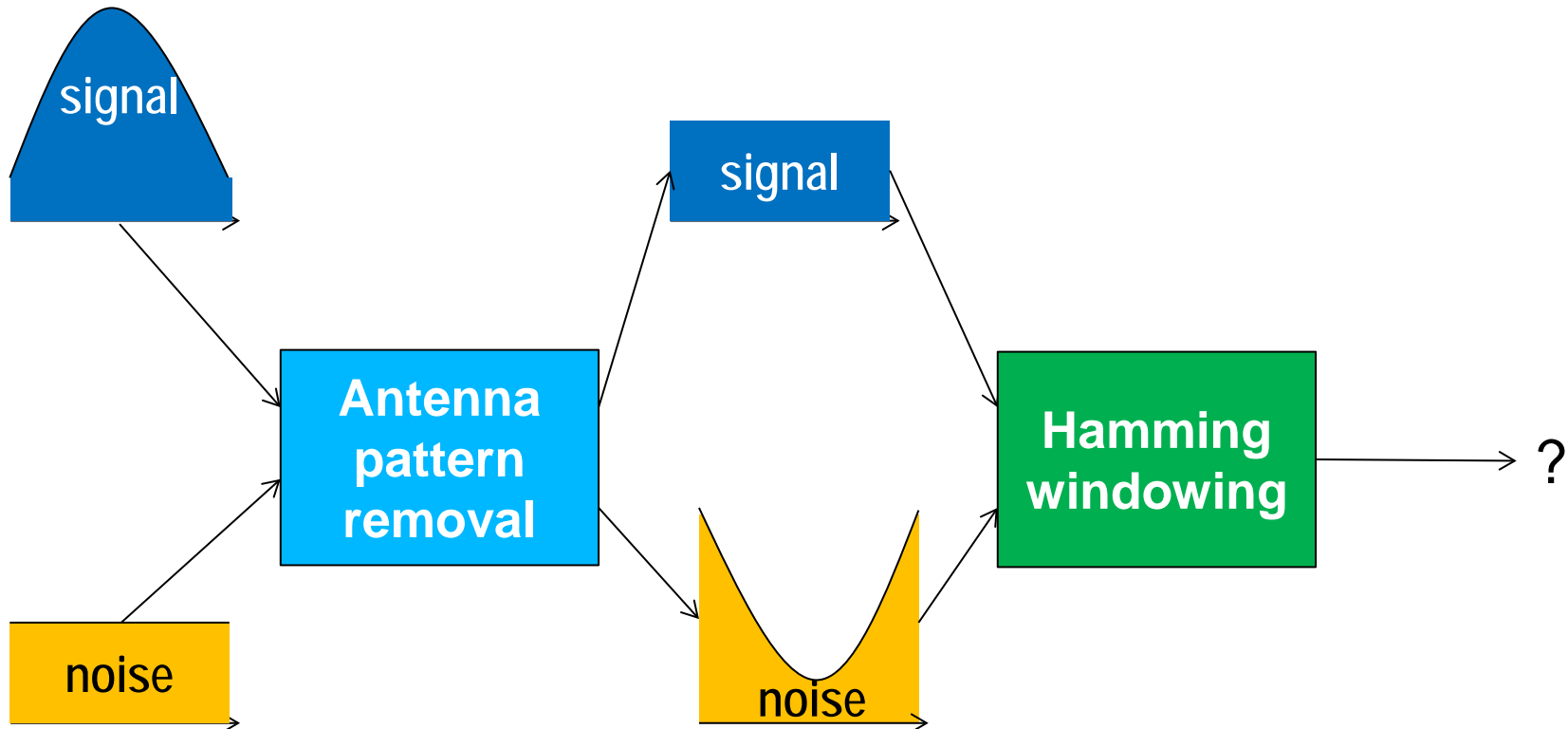




## White and Colored Noise



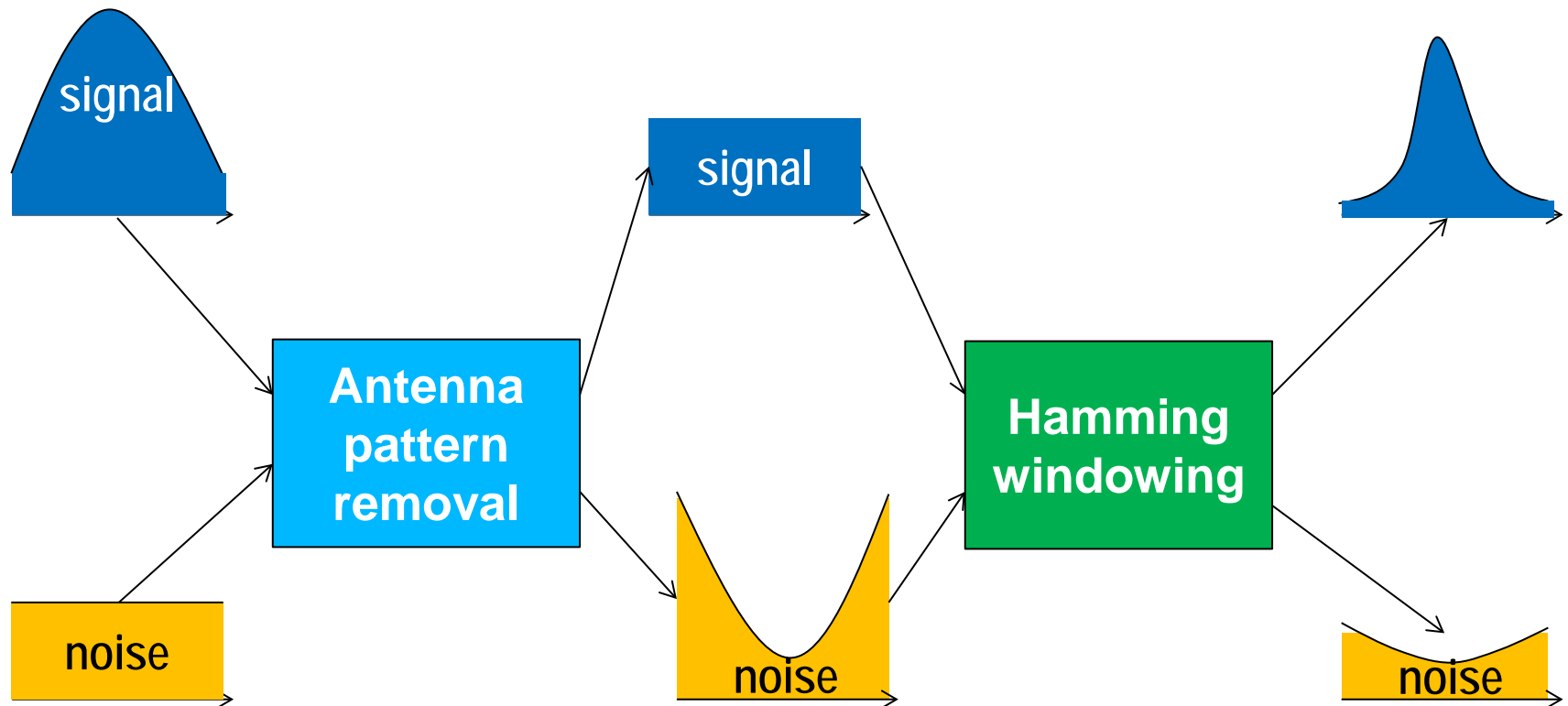
## White and Colored Noise



$$noise\ gain_{APR} = \frac{\int (N(f) \cdot A_{APR}(f))^2 df}{\int N(f)^2 df}$$



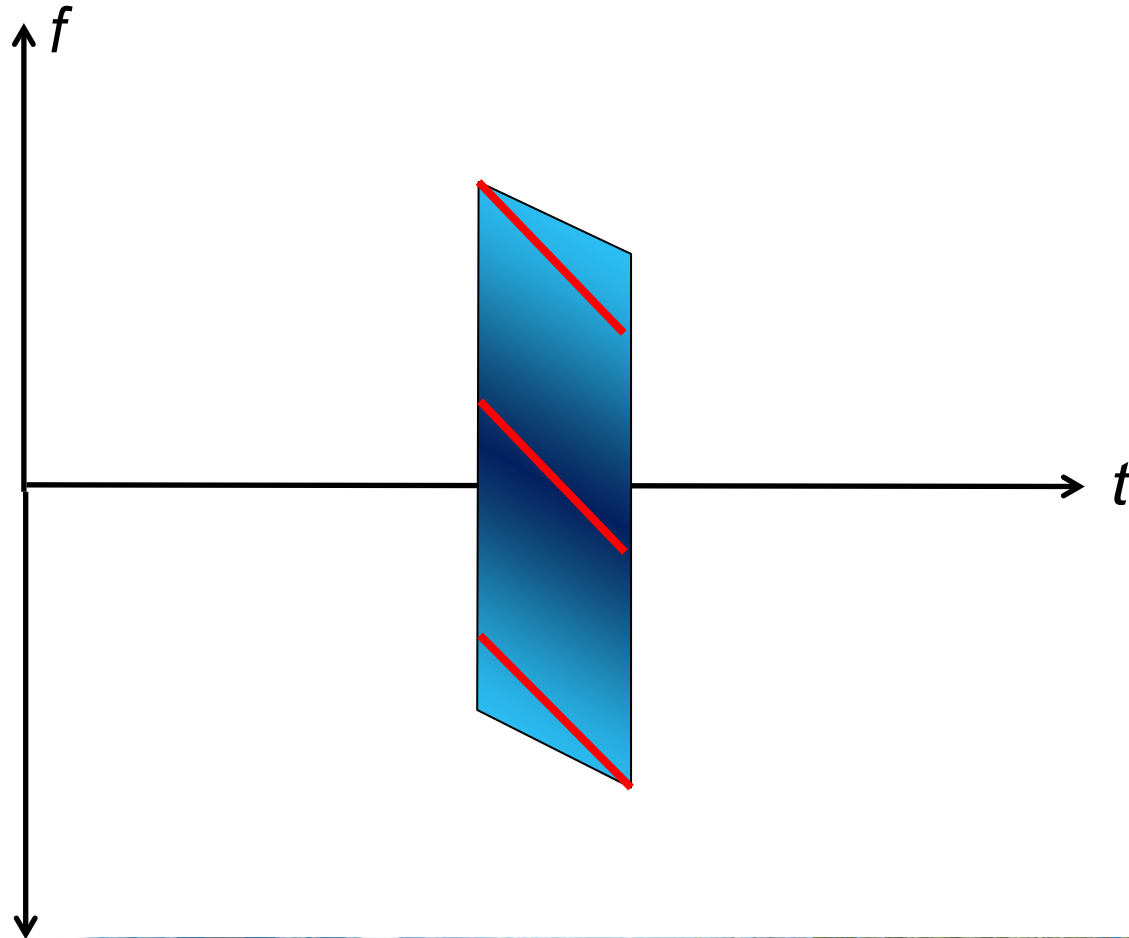
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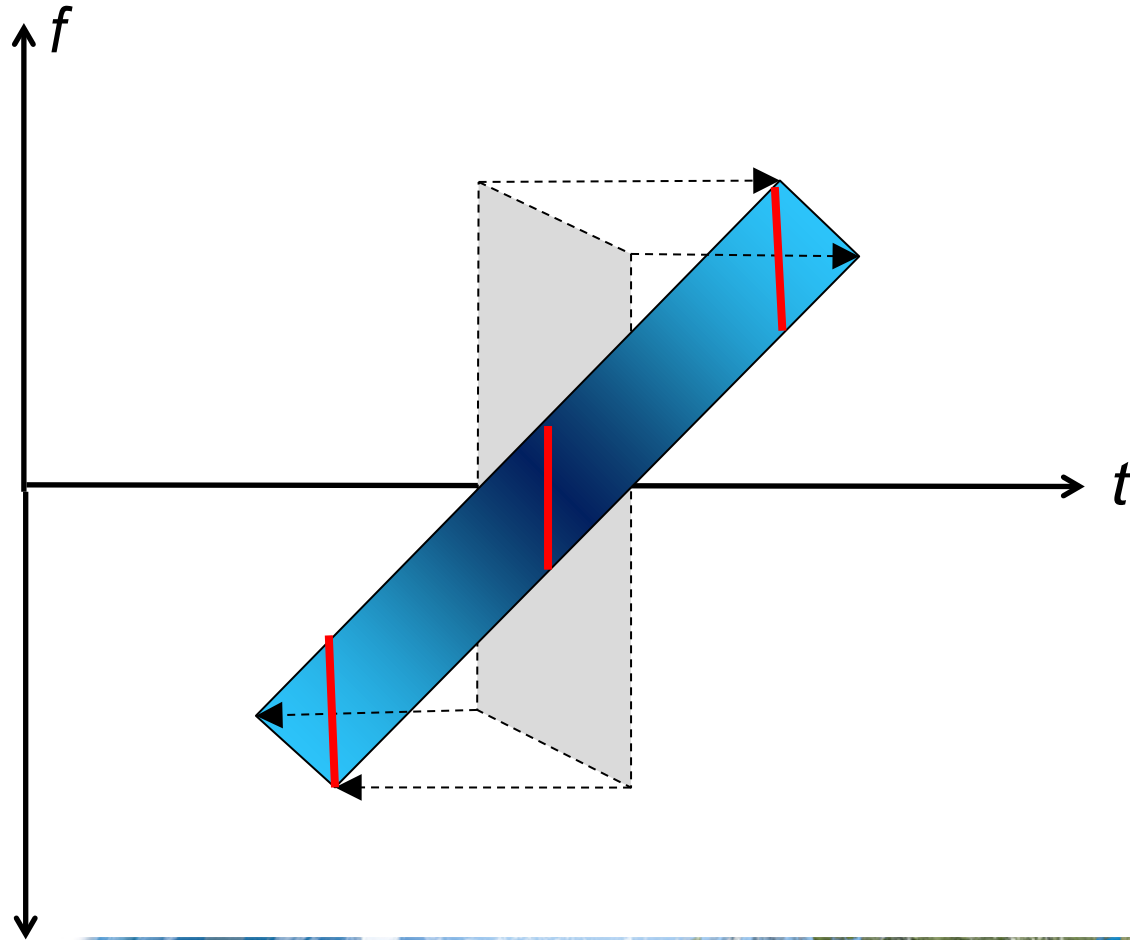
$$noise\ gain_{HW} = \frac{\int (N(f) \cdot \mathbf{A}_{APR}(f) \cdot A_{HW}(f))^2 df}{\int (N(f) \cdot \mathbf{A}_{APR}(f))^2 df}$$



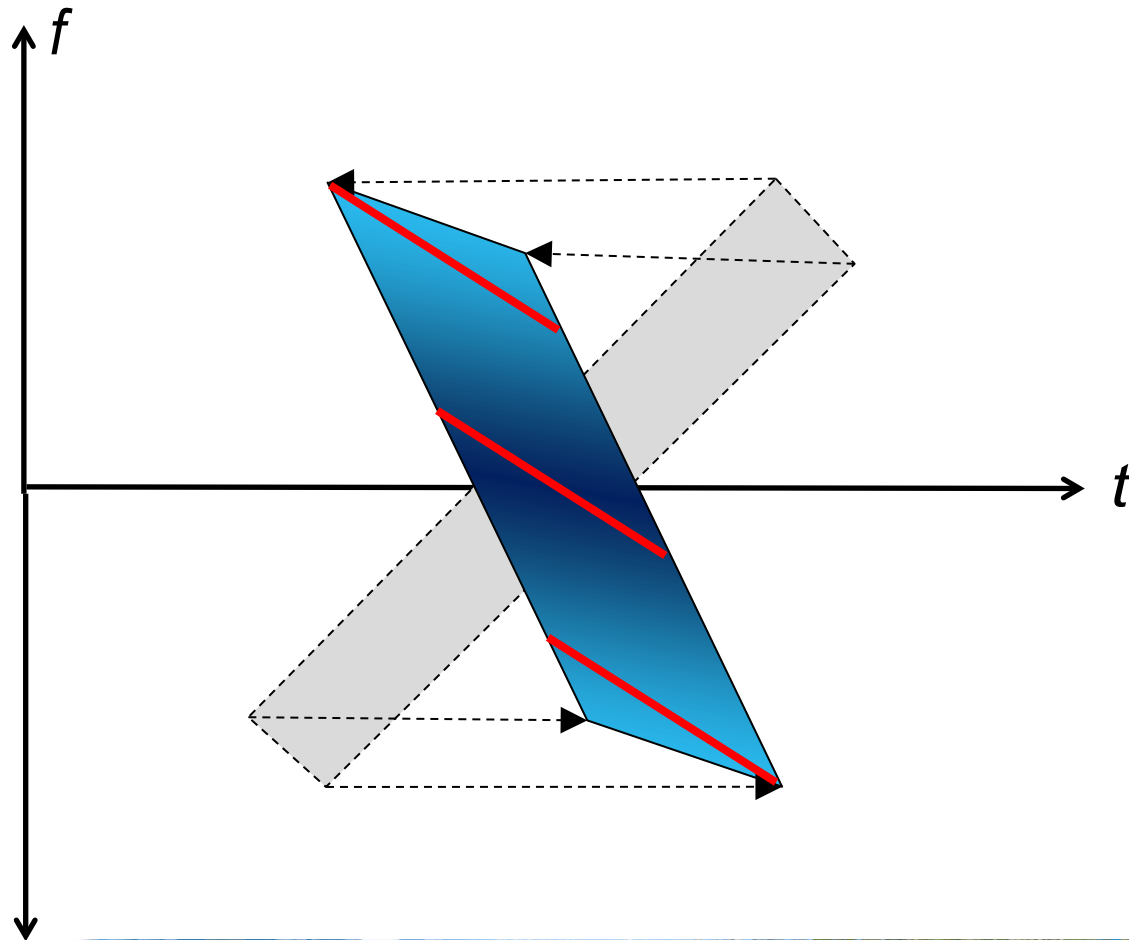
# Tracking the Signal through the SAR Processor (Range Compressed Data)



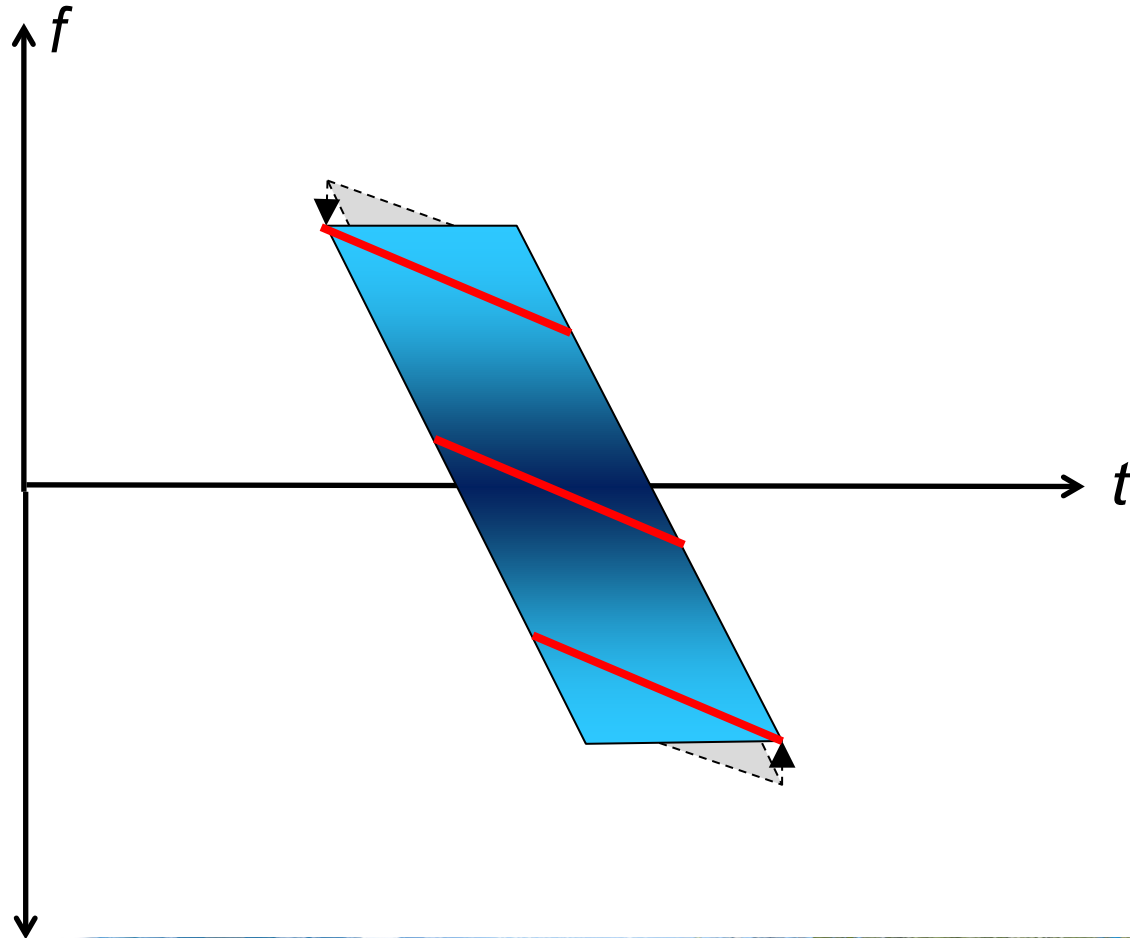
# Tracking the Signal through the SAR Processor (Azimuth Compression)



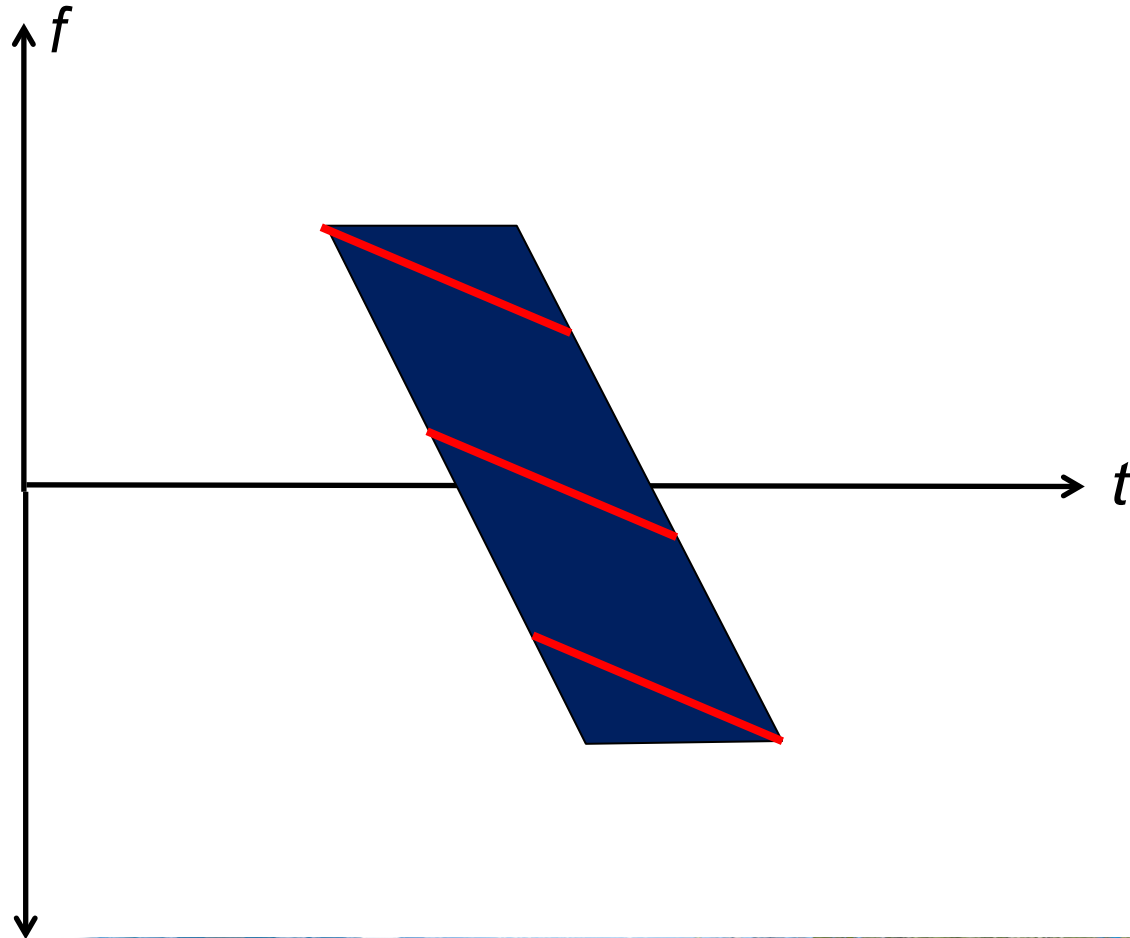
# Tracking the Signal through the SAR Processor (Azimuth Scaling)



# Tracking the Signal through the SAR Processor (Deramping for Antenna Pattern Removal)

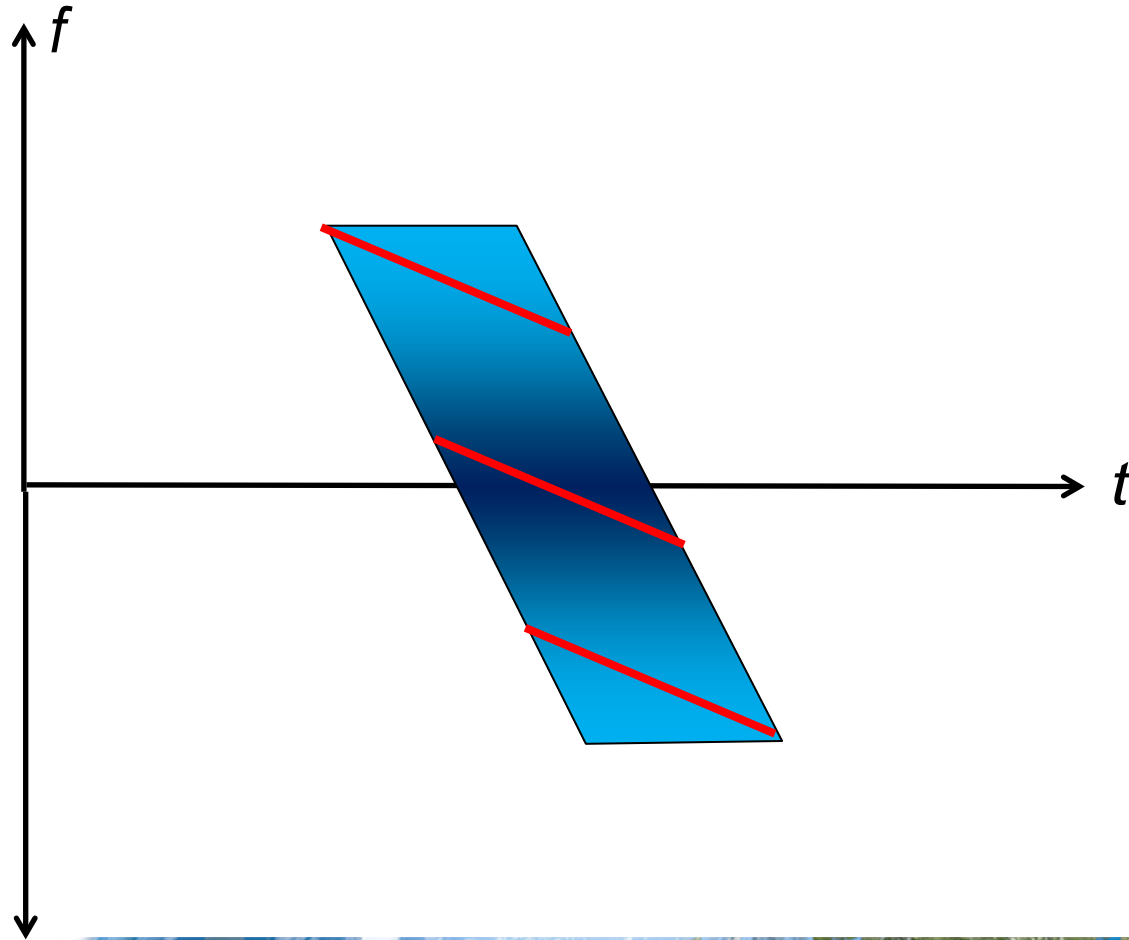


# Tracking the Signal through the SAR Processor (Antenna Pattern Removal)

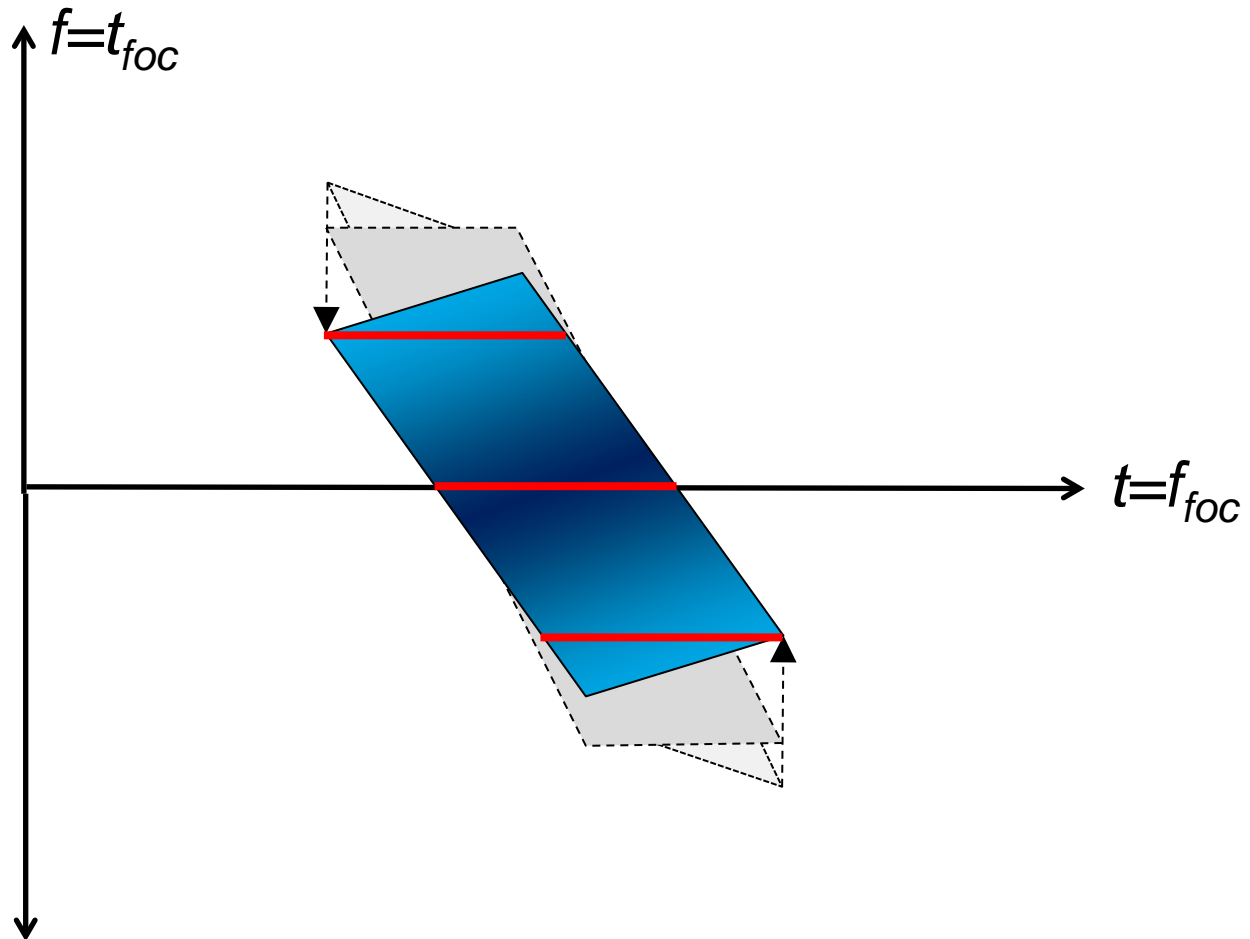




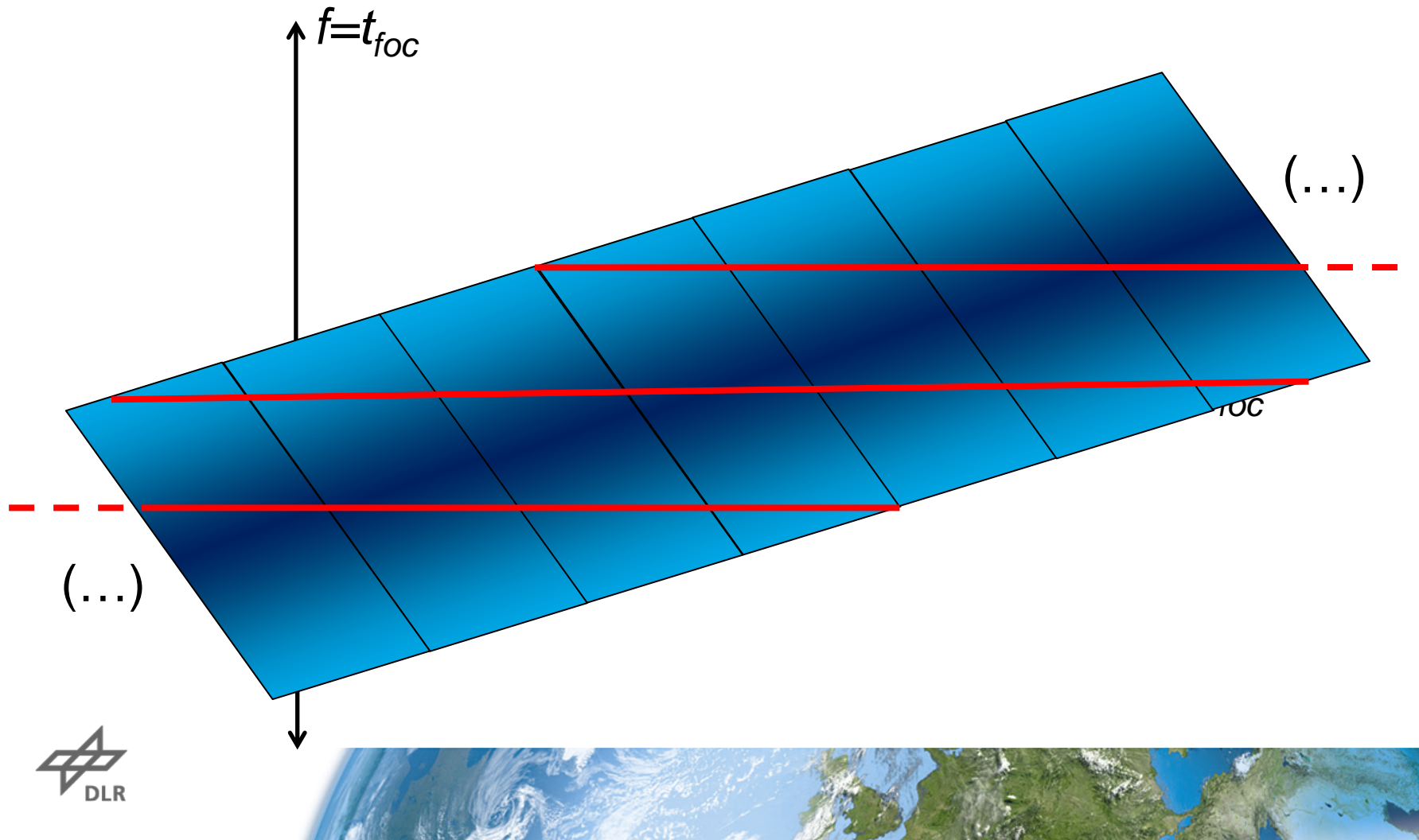
# Tracking the Signal through the SAR Processor (Multiply by Hamming Window)



# Tracking the Signal through the SAR Processor (Residual Deramping)

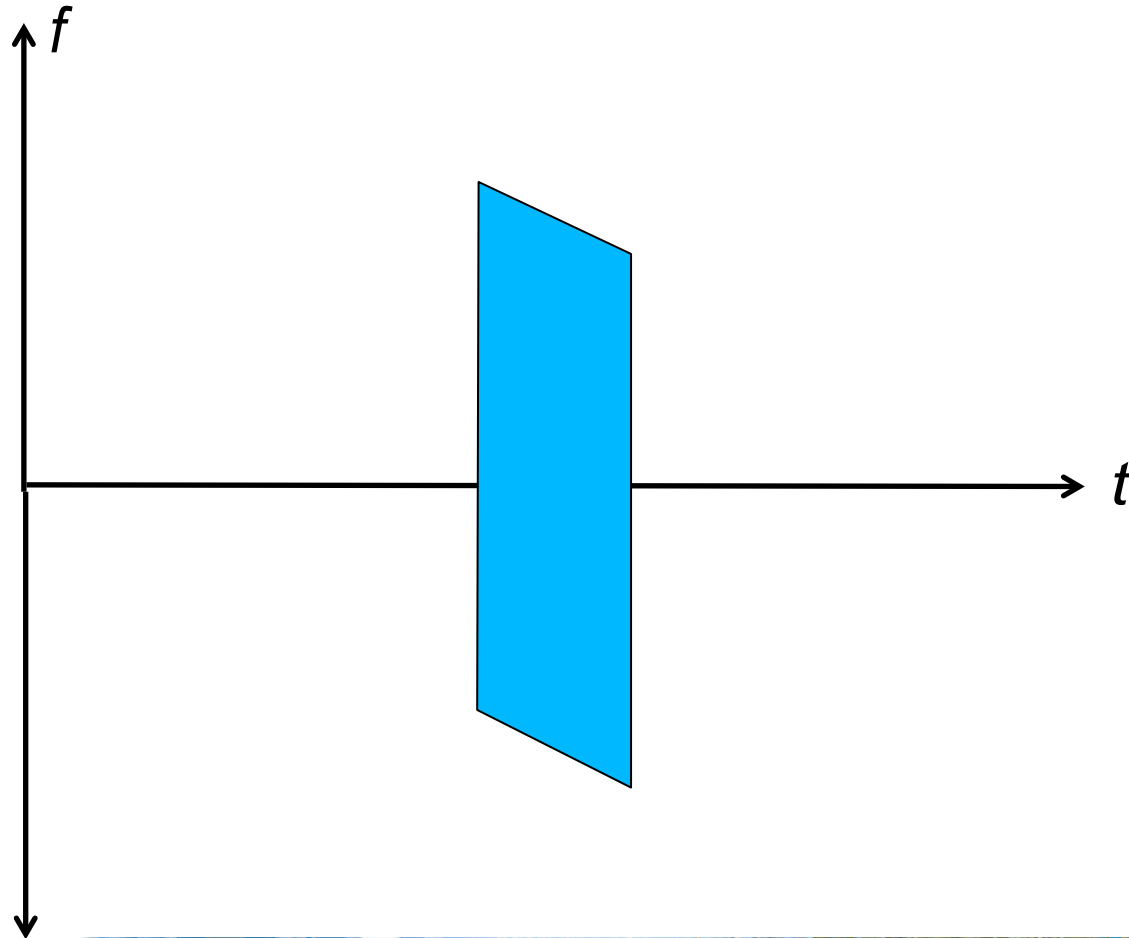


# Tracking the Signal through the SAR Processor (Recombination)

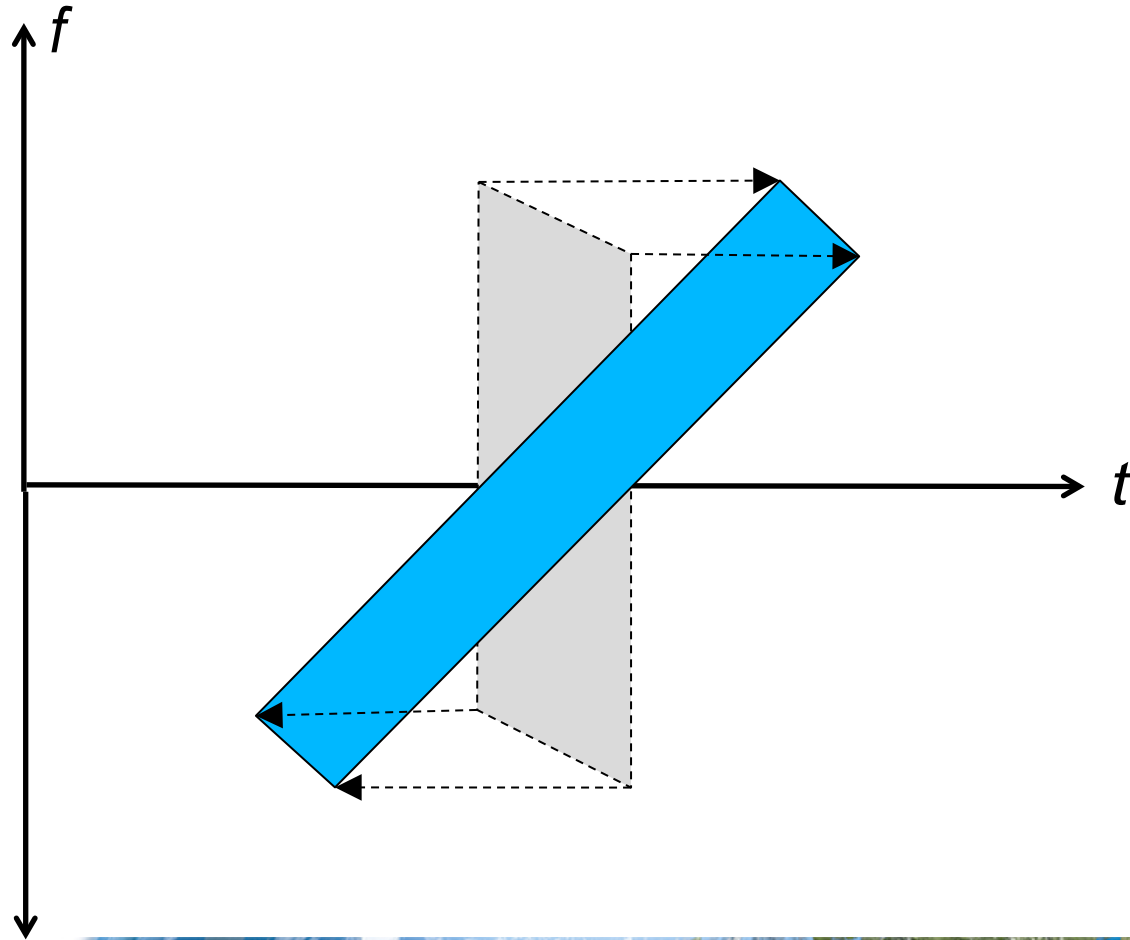




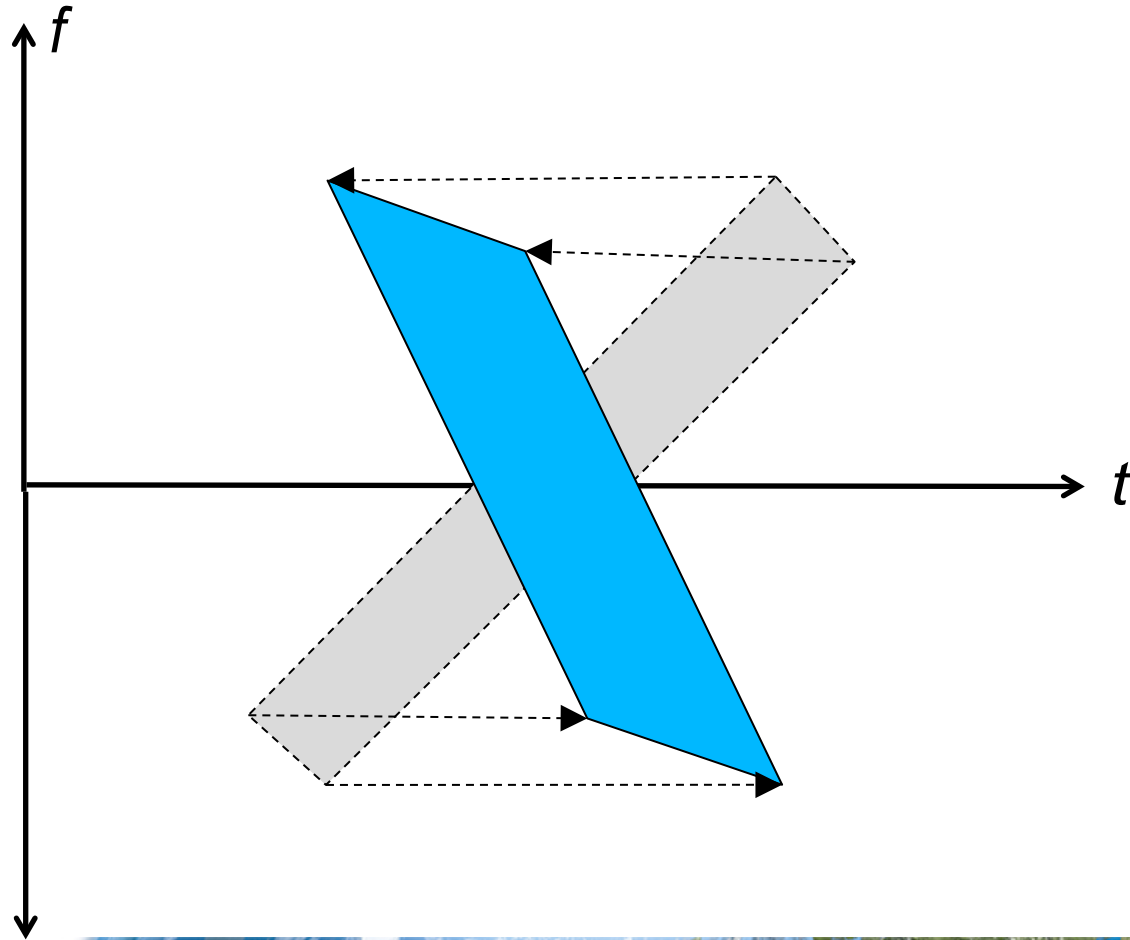
# Tracking the Noise through the SAR Processor (Range Compressed Data)



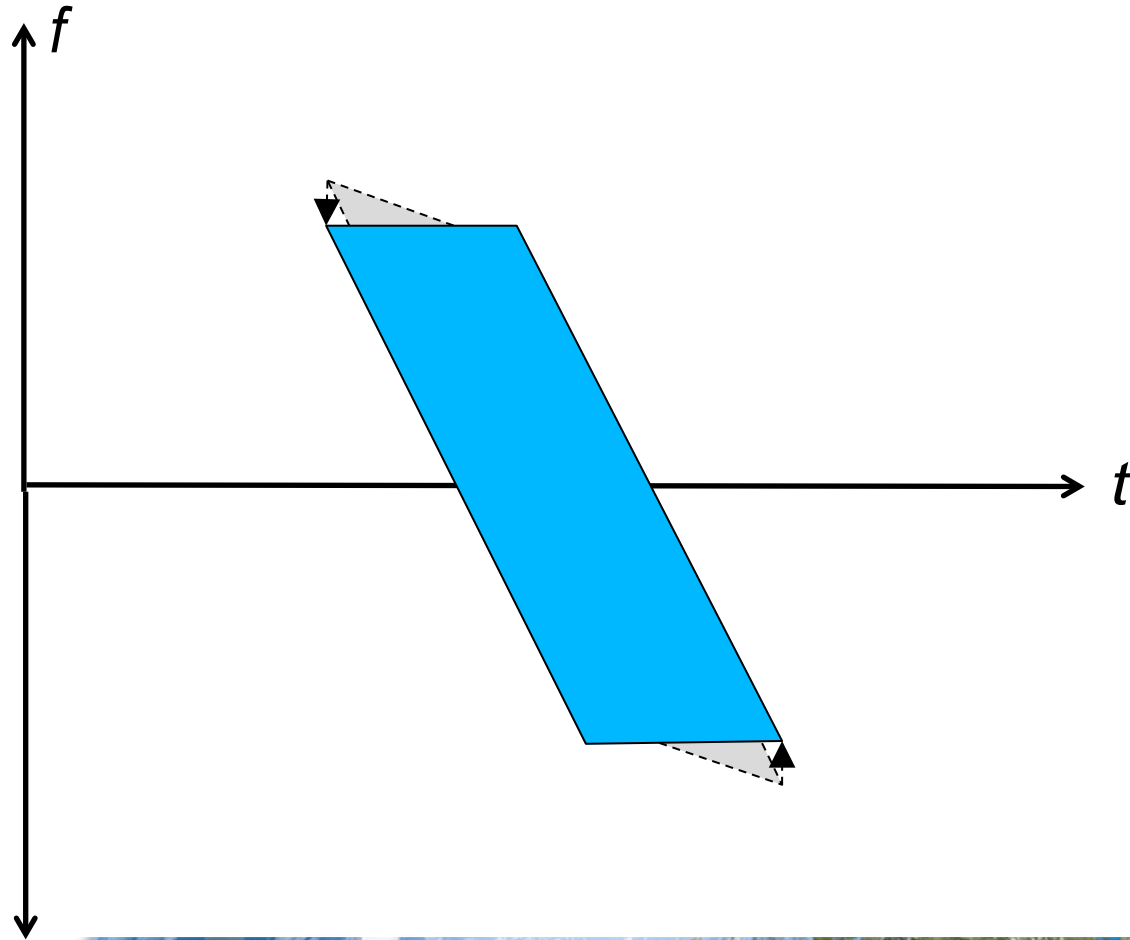
# Tracking the Noise through the SAR Processor (Azimuth Compression)



# Tracking the Noise through the SAR Processor (Azimuth Scaling)

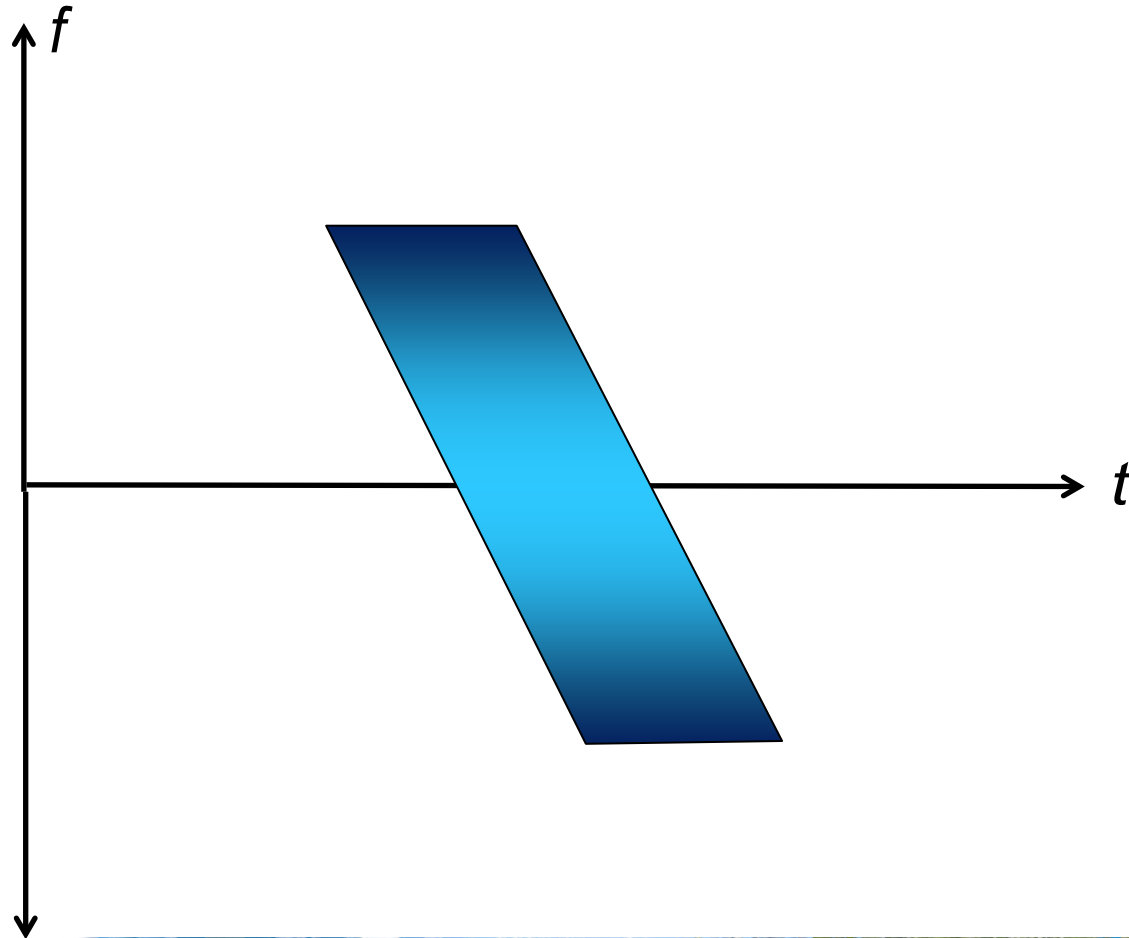


# Tracking the Noise through the SAR Processor (Deramping for Antenna Pattern Removal)

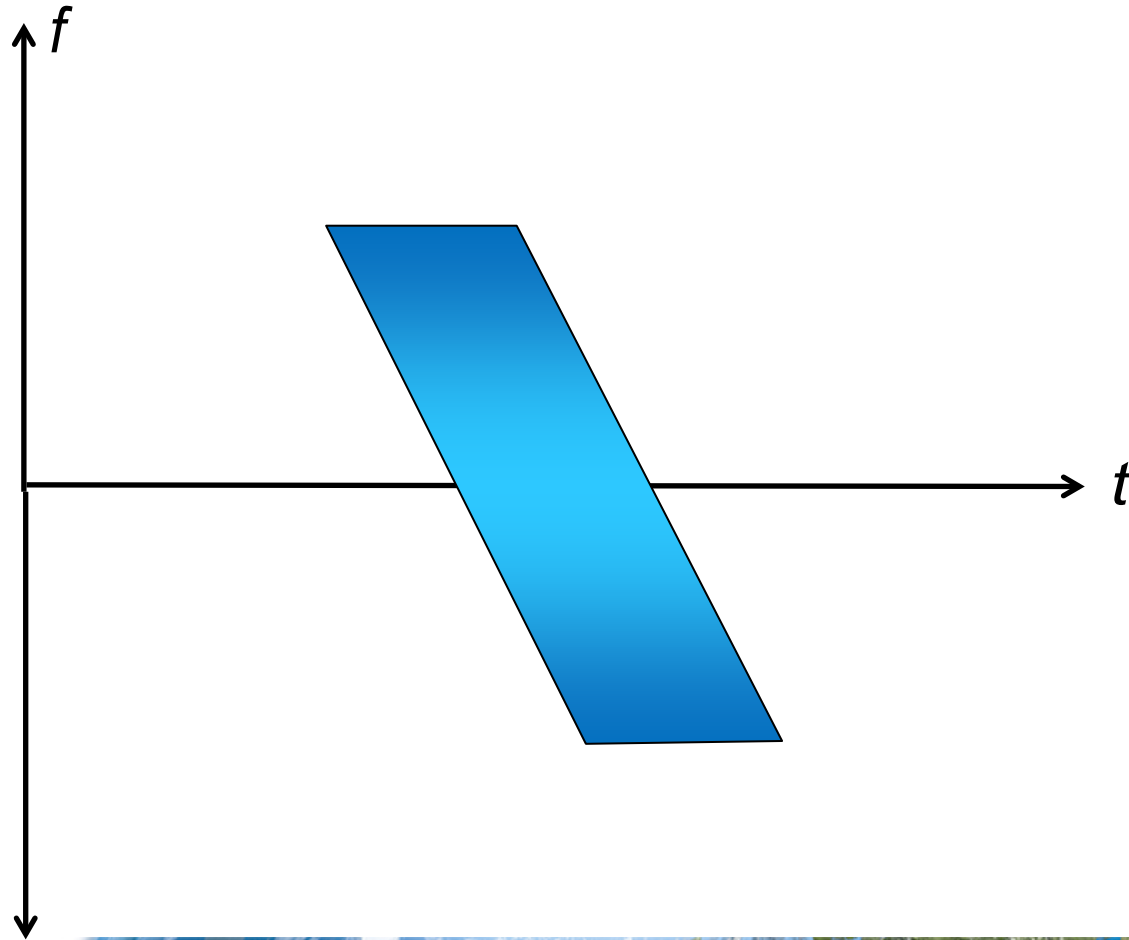




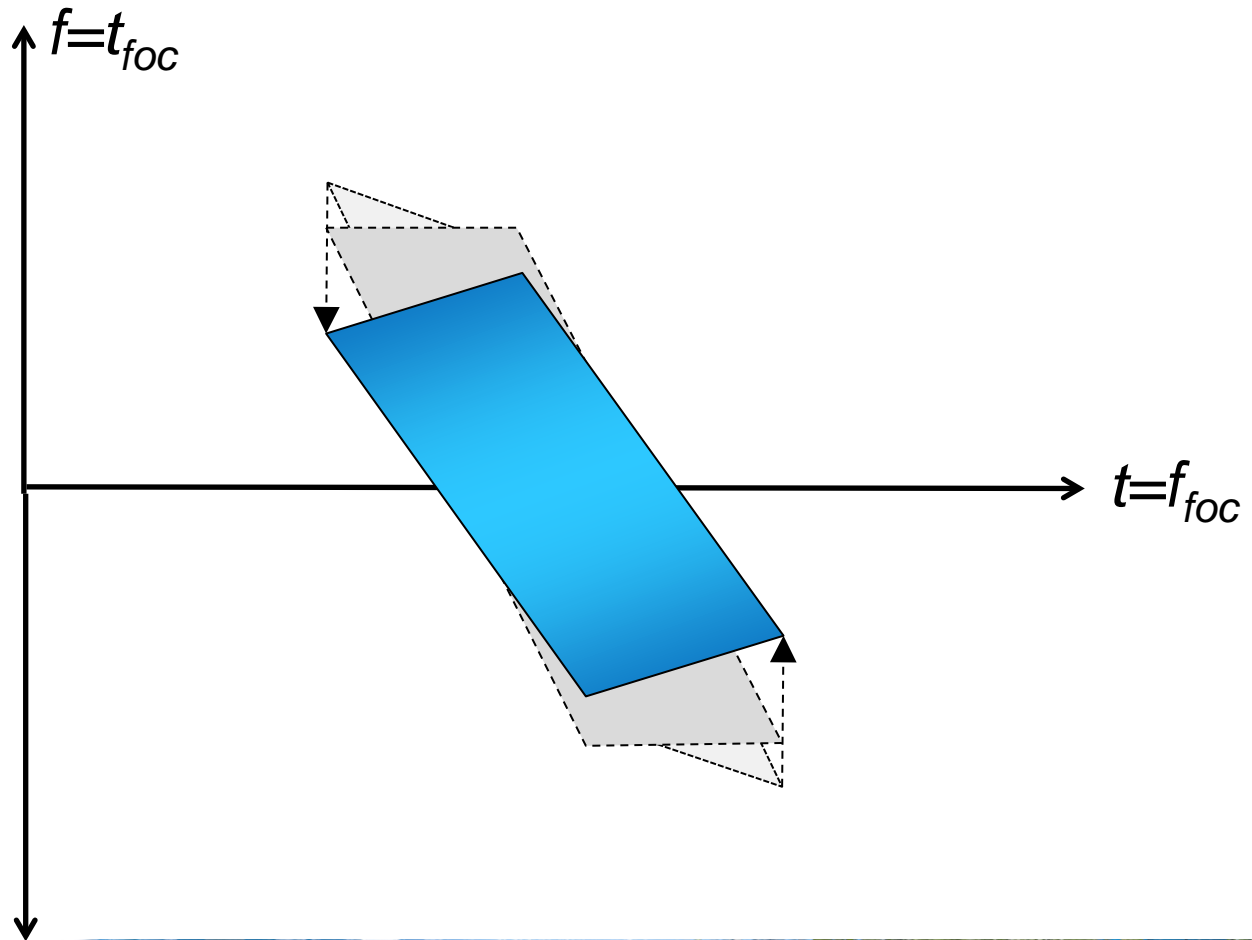
# Tracking the Noise through the SAR Processor (Antenna Pattern Removal)



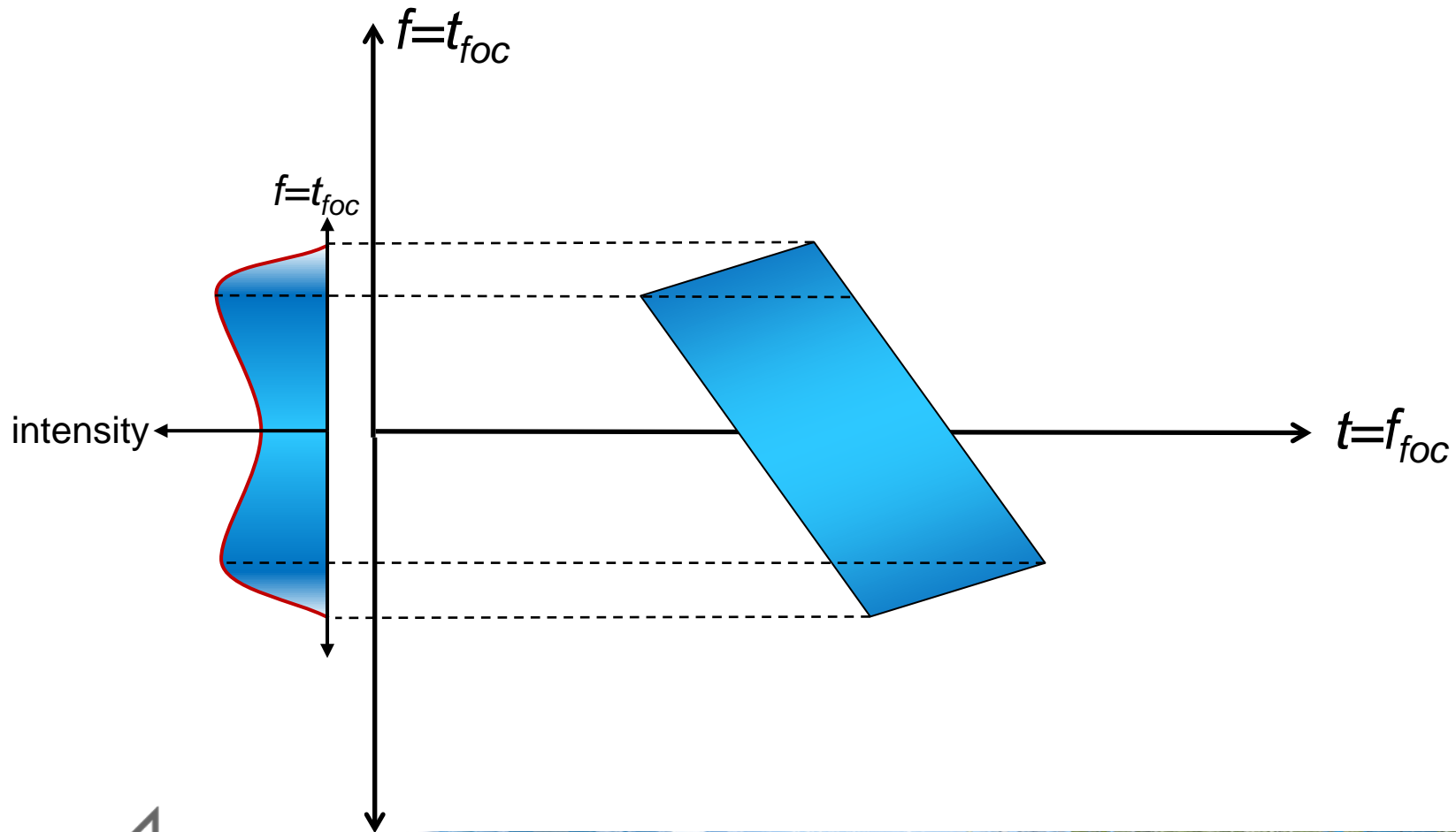
# Tracking the Noise through the SAR Processor (Multiply by Hamming Window)



# Tracking the Noise through the SAR Processor (Residual Deramping)



# Tracking the Noise through the SAR Processor (Image in SPECAN Domain)



# Conclusions

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  - Don't trust that white noise remains white after a filter step!



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- Signal and noise gain might differ according to their different spectral properties
- Keep in mind that in the processing chain, the input signal and noise is already changed by the predecessor steps
  - Don't trust that white noise remains white after a filter step!

## - A correct processor normalization is indispensable:

- It is required from the first days of the mission!



*Thank you for  
your attention!*



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